Matematisk-fysiske Skrifter
udgivet af
Det Kongelige Danske Videnskabernes Selskab
Bind **3**, nr. 3

Mat. Fys. Skr. Dan. Vid. Selsk. **3**, no. 3 (1967)

DEVELOPMENT OF IMPERSONAL TRANSIT CIRCLE METHODS

BY

SVEND LAUSTSEN

København 1967
Kommissionær: Munksgaard

DET KONGELIGE DANSKE VIDENSKABERNES SELSKAB udgiver følgende publikationsrækker:

THE ROYAL DANISH ACADEMY OF SCIENCES AND LETTERS *issues the following series of publications:*

	Bibliographical Abbreviation
Oversigt over Selskabets Virksomhed (8°) *(Annual in Danish)*	Overs. Dan. Vid. Selsk.
Historisk-filosofiske Meddelelser (8°)	Hist. Filos. Medd. Dan. Vid. Selsk.
Historisk-filosofiske Skrifter (4°) *(History, Philology, Philosophy, Archeology, Art History)*	Hist. Filos. Skr. Dan. Vid. Selsk.
Matematisk-fysiske Meddelelser (8°)	Mat. Fys. Medd. Dan. Vid. Selsk.
Matematisk-fysiske Skrifter (4°) *(Mathematics, Physics, Chemistry, Astronomy, Geology)*	Mat. Fys. Skr. Dan. Vid. Selsk.
Biologiske Meddelelser (8°)	Biol. Medd. Dan. Vid. Selsk.
Biologiske Skrifter (4°) *(Botany, Zoology, General Biology)*	Biol. Skr. Dan. Vid. Selsk.

Selskabets sekretariat og postadresse: Dantes Plads 5, København V.

The address of the secretariate of the Academy is:

Det Kongelige Danske Videnskabernes Selskab,
Dantes Plads 5, Köbenhavn V, Denmark.

Selskabets kommissionær: MUNKSGAARD's Forlag, Prags Boulevard 47, København S.

The publications are sold by the agent of the Academy:

MUNKSGAARD, *Publishers,*
47 Prags Boulevard, Köbenhavn S, Denmark.

Matematisk-fysiske Skrifter
udgivet af
Det Kongelige Danske Videnskabernes Selskab
Bind **3,** nr. 3

Mat. Fys. Skr. Dan. Vid. Selsk. **3,** no. 3 (1967)

DEVELOPMENT OF IMPERSONAL TRANSIT CIRCLE METHODS

BY

SVEND LAUSTSEN

København 1967
Kommissionær: Munksgaard

CONTENTS

Synopsis

The transit circle at the new observatory of the Copenhagen University, which is situated near the village of Brorfelde, has been equipped with a photographic micrometer. In order to achieve sufficient accuracy for the tracking of stars during meridian passage, an electronic, digital, velocity-generating device has been developed. A high degree of automation has been utilized in the timing and control of observations. Originally a photographic method of circle reading was applied, but from December 1965 a new-developed photoelectric recording of the circle has been in use. The method of photoelectric scanning is applied, and the signal output is punched in digital form suitable for electronic data handling. The accuracy of this method has been proven to be very high. The observational results are mentioned; night-to-night comparisons of differential observations give the internal mean errors $\varepsilon_\alpha \cdot \cos\delta = 0^s\!.013$ and $\varepsilon_\delta = 0''\!.20$ for a single observation of a star brighter than the visual magnitude 8.5 and somewhat larger errors for fainter stars.

PRINTED IN DENMARK
BIANCO LUNOS BOGTRYKKERI A/S

1. Introduction

In commemoration of the 300th anniversary of the birth of OLE RØMER, September 25, 1944, the Copenhagen University received a grant for a new transit circle from the Carlsberg Foundation. This grant was supplemented by a second one on the occasion of the 400th anniversary of the birth of TYCHO BRAHE, December 14, 1946.

In 1950, the foundation for the transit circle was laid on a hill outside the village of Brorfelde which is situated 60 kilometres west of Copenhagen. This place was selected as site for the new observatory which had been planned by Professor BENGT STRÖMGREN.

The transit circle, manufactured by GRUBB PARSONS, was mounted on the site in 1953. In 1959 the instrument was fitted with a photographic micrometer, which had been constructed in the observatory workshop according to specifications given by STRÖMGREN. The results of observations in 1960–61 demonstrated clearly the usefulness of the impersonal method and, furthermore, they proved it worthwhile to build a new micrometer on the basis of the experience gained from observations with the first version. In 1961, it was, therefore, decided to initiate design and construction of a new micrometer and control units. This apparatus and the newly developed device for photoelectric recording of circle setting are described in the present paper; and the basic principles for the reduction procedure and some observational results are mentioned here previous to the first observational catalogue.

2. Original Design of the Instrument

The construction of the reversible seven-inch transit circle for the Brorfelde Observatory was completed by GRUBB PARSONS in 1952; the instrument was mounted on the site the following year. In most respects, the instrument is similar to the transit circle of the Royal Greenwich Observatory. A general view of the instrument is shown in Fig. 1.

The telescope has a focal length of 2665 mm, the clear aperture being 178 mm. The object glass is a separated doublet corrected for the visual wave-lengths. Each of the two identical tube halves constitutes one single iron casting. The cross section of the tube has the form of an oval which approaches a circle at the outer ends. The inner side of the tube is ribbed, whereas its outer side is fitted with radiation shields made of nickel-plated copper.

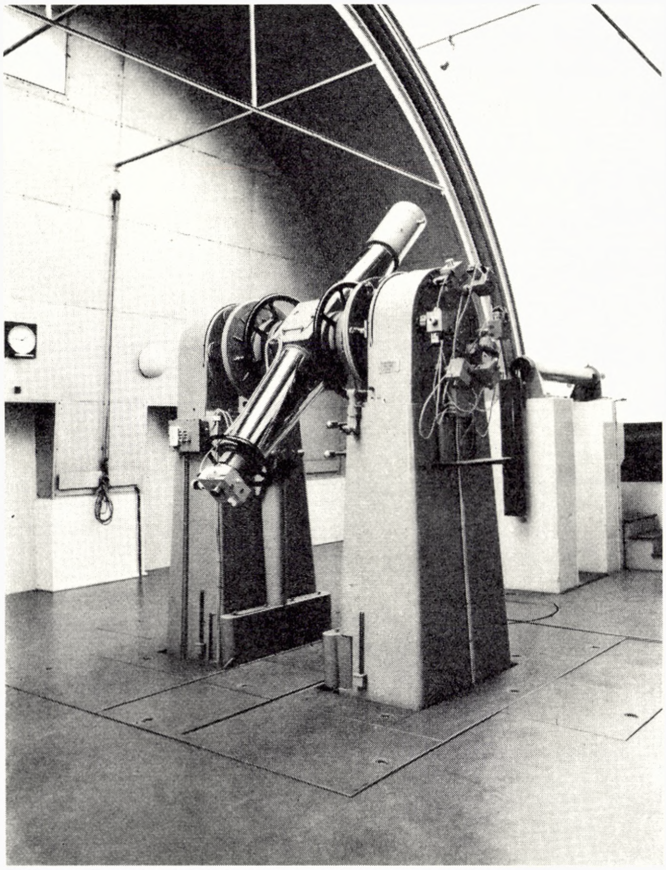

Fig. 1. The transit circle as seen from the northwest.

The axis of the transit circle comprising the central cube and the two circular hollow pivot extensions is one unit made from a single iron casting. A handwheel, weight-relief ball bearing, pivot ring, and end thrust pad are mounted on each axis half. One end has, moreover, an axis clamp with a slow-motion adjusting screw and a cast iron housing for the declination circle. These latter parts are balanced by a weight at the other end of the axis. The pivots, which are hollow cylinders made of flint hard steel shrunken on the ends of the axis, have a heavy chromium plating which has been ground, lapped and polished so as to render the form of each pivot cylindrical within an accuracy of half a micron and their diameters identical within the limits of one micron (according to the specifications given by Grubb Parsons). The pivots have diameters measuring 130 mm. Each of them rests upon a bearing made of two bronze shoes spaced 90 degrees apart. The bearing surface of the shoes measures 19 mm along the axis and 32 mm on the circumference. The distance between the two sets of bronze shoes, i.e. the effective length of the axes, is 1360 mm.

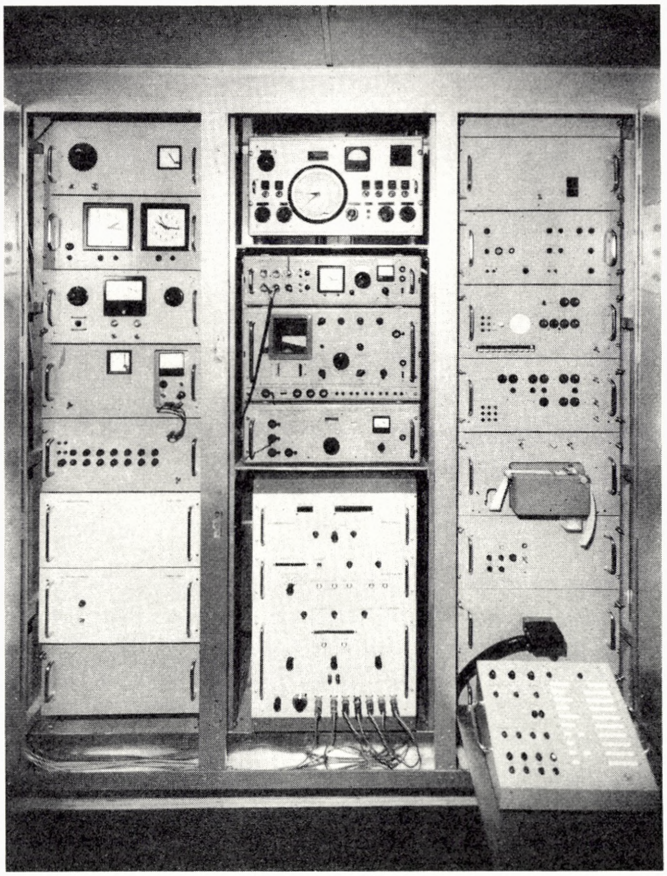

Fig. 2. Transit circle control panels. To the right is seen the photographic micrometer control panel with the following units from below: The keyboard which is connected with a keyboard control unit, the oscillator and the divider unit of the velocity-generating device, the Friden tape punch, the observation timer, the master control unit including the secδ-storage and the punch control, the flash lamp control unit and the declination camera control and uppermost the motor unit including pulse-to-step translator, stepping motor, and torque transmitter. The middle section contains from the top: radio receiver, quartz clock, time signal oscillograph, and sidereal time converter. The lower part of the middle section contains the data logging system for the recording of the circle setting; the Teletype punch is not shown in the picture. The left rack contains various power supply units and the motor control unit for the photoelectric micrometers.

The weight relief of the instrument is provided by two thrust columns arranged beneath the axis and connected with simple lever systems. A roller chain at the top of each column gives a well-distributed support to the telescope axis through the large ball bearing on each half of the axis. The residual load upon each pivot is adjustable by means of bob weights to be shifted along rods with scales. The load used is about 10 kg on each pivot.

The two piers, which support the bearings for the axis of the instrument and the microscopes for circle reading, rest upon a common bedplate below floor level. Both bedplate and piers are hollow boxes made of cast iron, and the piers are protected externally by a layer of insulation and a skin of aluminium sheeting. The bedplate is carried by a strong cast iron base plate, in relation to which it is adjustable for level. The base plate, on the other hand, is adjustable in azimuth within a cast iron base trough. The trough is filled with oil to prevent rusting of the screws which control the level and the azimuth adjustments. The base trough is set upon a solid concrete block extending to a depth of 4.3 metres.

The instrument is equipped with a glass circle, on which etchings filled with paint mark the graduations. The glass disc, which is 9.4 mm thick, has a diameter measuring 724 mm and a central cutting with a diameter of 311 mm. The divisions are placed on the disc, with a division line for every 5 minutes of arc, round a circle with a diameter of 610 mm. Accordingly, the linear distance between the lines is 0.443 mm. The width of the division lines is approximately 0.017 mm. The weight of the disc rests on 18 points of support, which are arranged with 12 points in a ring outside the graduated circle and 6 points in a ring within this circle. At each point of support, a lever with ball end engages into a small hole bored in the glass disc, which is then given a suitable vertical force by attaching a counterweight to the other end of the lever.

Six circle reading microscopes are mounted in cylindrical openings at the top of the west pier 60 degrees apart around the declination circle. Each microscope is provided with a lamp housing at the front end. The focal length of the objectives is 102 mm, and a magnification of 7.9 is used. Until December 1965 a photographic registration of the circle setting was used, but since then a photoelectric technique of recording has been applied. A description of the two methods is given in Sections 7 and 8. A control of the circle setting is secured by means of a low-power microscope, the field of which covers more than one degree of the circle. Each degree of the circle graduations is numbered. The low-power microscope is equipped with a camera for photographic recording; this camera was applied for rough reading also after the introduction of the photoelectric technique for the accurate reading.

Two collimators with focal lengths of 1800 mm and apertures of 178 mm are installed in the transit circle room, resting upon concrete blocks extending through the floor, one to the north and one to the south of the instrument. Below floor level a nadir mercury trough is mounted upon the bedplate of the instrument.

The pavilion for the transit circle consists of a base of concrete walls up to three metres above ground level, which is covered by a semi-cylindrical metallic dome. The floor is two metres above ground level and measures eight metres from the east to the west and eleven metres in the north-south direction; the interior diameter of the semi-cylinder is also eleven metres. The dome is constructed of steel with outer and inner coverings of iron plates. Precautions have been taken to facilitate ventilation by the natural draft in the air space between the two coverings, whereas no thermal insulation of the pavilion has been provided. The dome has two shutters which can slide backwards to the east and the west, thereby yielding an aperture of 2.8 metres along the meridian. An annex on the east side contains entrance, staircase and a room with time-service and control panels. The clock and all other electronic units are kept in an insulated cupboard under thermal control in order to minimize the transfer of heat from this equipment to the transit circle room. A general view of the electronic accessories is shown in Fig. 2.

3. The Photographic Micrometer

In the photographic micrometer the transit of the stars is recorded on a photographic plate, which moves in the focal plane of the telescope in accordance with the velocity of the stars under observation. The photographs of the micrometer represented in Figs. 3 and 4 show the design of this equipment.

The body of the micrometer is a box 22 cm × 22 cm × 10 cm with a flanged socket for attaching the micrometer to the instrument tube and for adjustments, which is seen in Fig. 3. To the right is seen the torque receiver for the movement of the plate and above it is the shutter coil. These constituents are placed outside the box in order to prevent possible thermal disturbances.

The exposure is controlled by an ordinary Compur shutter in connection with a clock-controlled shutter coil. Generally, the exposure time for stars is 20 seconds. It is possible, however, to extend the exposure time to 40 seconds in order to reach fainter stars.

After passing the shutter, the light beam from the objective must penetrate a glass disc, which is divided into four sections with different light absorbing powers, so as to form a series of neutral filters to weaken the light of the brighter stars. The first filter section consists of clear glass, while the three other sections are aluminized so as to absorb $1\overset{m}{.}5$, $2\overset{m}{.}5$ and $4\overset{m}{.}0$ respectively. The glass disc is mounted in a carriage guided by a cylindrical way and rail attached to the circular cover plate towards the tube. The filter selection is performed by a knob outside the box, which is visible to the left in Fig. 3; pilot lamps indicate which filter is in use.

The light beam now proceeds through a wedge-shaped tube, the wide end of which is visible in Fig. 3. The graticule* is fixed at the narrow end of the tube. The graticule is ruled on a filter glass of type GG14 and of 18 mm × 10 mm size. The combined graticule and yellow filter glass plate is separated from the photographic plate by about 0.2 mm of air. The graticule plate is opaque except for three clear openings. A rectangular opening 11 mm × 2.5 mm allows the light beam passing through the tube to reach the photographic plate. Two tiny circular openings with a 0.03 mm diameter, one on each side of the rectangular opening and at a mutual distance of 4 mm, serve as a reticle. During the observations the photographic plate is momentarily exposed to flash light through the small openings from which circular marks are exposed on the photographic plate. These are the reference marks in the subsequent measuring of the plate, and therefore particular precautions have been

* The graticule was manufactured by J. Heidenhain by means of a Diadur process.

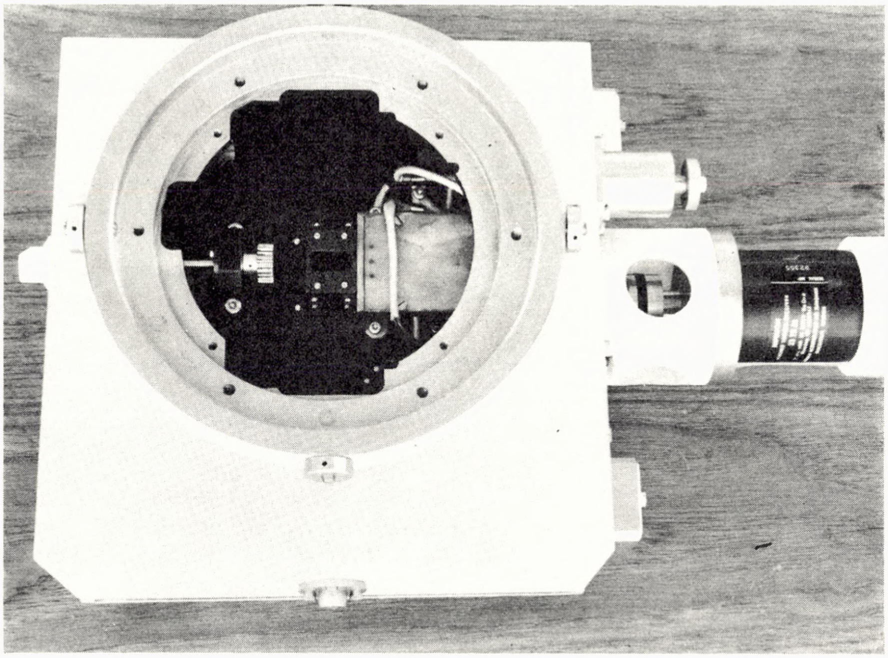

Fig. 3. The micrometer viewed from the tube side. The circular cover plate with the shutter and the glass filter has been removed in this picture.

taken to ensure a rigid mounting of the graticule plate and the flash assembly at the tube of the instrument.

Two carriages in the micrometer allow the photographic plate to be moved translationally in two perpendicular directions, while it is held parallel to the graticule plate at the constant distance. The primary carriage is the right ascension slide, and on this a secondary carriage with a plate holder is mounted. The right ascension slide is guided by a cylindrical way and a guide rail, which are both attached to the micrometer box. The movement of the slide along the guides is controlled by the screw through a non-backlash nut, which is attached to the slide. The screw is turned by a torque receiver through a gear with the ratio 5:1. During the exposure the right ascension slide moves with a uniform speed, which corresponds to a high degree of accuracy with the speed of the stellar image in the focal plane.

The size of the plates is 6 cm × 9 cm, and although the section used is only 3 cm × 6 cm, each plate holds about 70 observations. The observations are arranged in 12 horizontal lines on the plate; in order to achieve this easily, the plate holder has been constructed as a carriage with an automatic line-spacing mechanism. When a line is filled, the right ascension slide returns with high speed, and during the last part of this return the plate holder carriage is shifted 5 mm to allow observations on a new line. The plate holder carriage is guided in the same way as the right ascension

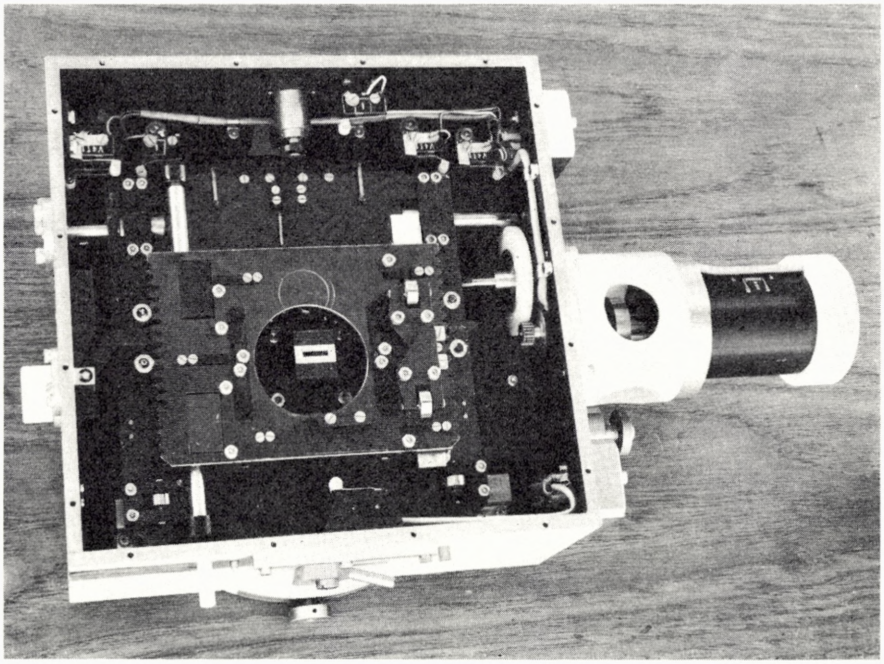

Fig. 4. The micrometer with the back cover removed.

slide, except that the guides for this carriage are of course fixed to the right ascension slide.

A few stop switches, all of which are shown in Fig. 4, are necessary in order to control the movement of the two slides. The activation of a switch might, however, have a perceptible effect on the slides. In order to eliminate this source of error, the observations are carried out in such a way that the slides do not have contact with any switch or any mechanical part of the line-spacing mechanism during an observation.

Fig. 1 shows the micrometer mounted on the telescope tube. Two flexible cables connect the control panels with the micrometer. Loading and unloading are effected through the slit in the upper side of the micrometer box. During loading or unloading, the plate holder carriage must be in its uppermost position, and the right ascension slide must be in the middle position. The cylinder on the back cover of the micrometer box is filled with a drying agent. The cylinder may, however, be replaced by an eye piece for visual inspection, which is convenient when certain adjustments of the instrument are to be made. In this case a cross wire can be placed in the plate holder also.

4. Digital Velocity-Generating Device for the Right Ascension Slide

When a star crosses the meridian, its image in the focal plane of the instrument moves along a trace normal to the plane of the meridian. For a star with apparent declination δ, the velocity V is given by the expression:

$$V = V_e \cdot \cos\delta \quad \text{mm/sec} \tag{1}$$

Here the time unit is sidereal seconds, and V_e is the velocity of an equatorial star, which is equal to the scale value of the telescope. The horizontal scale reduction owing to atmospheric refraction is included in V_e, but apart from this slight correction the velocity V is unaffected by refraction.

The right ascension slide of the photographic micrometer is adjusted so as to move at right angles to the plane of the meridian according to the star images. During this motion, which is controlled by the micrometer screw, the slide constantly keeps the photographic plate in the focal plane of the telescope. Thus, with a uniform motion of proper speed, the plate will follow the star image accurately, and the star will appear as a small circular dot on the developed plate.

During the exposure of a star the guiding of the plate must be very accurate. The rate of motion should be uniform enough to allow deviations of the order of only 0.001 mm between an actual placing and a placing obtained by a perfectly uniform rate. The principal reason for this condition is that the reference marks are placed instantly upon the moving photographic plate.

Although the rate of motion must be quite uniform during the exposure, the rate adopted need not be completely equal to the speed of the star image. A deviation in speed of 0.1 per cent is quite acceptable. The scale value is measured to be 0.19384 mm/sec, and with the exposure time of 20 seconds the trace of an equatorial star will amount to nearly 4 mm. If a deviation of 0.1 per cent is tolerated, it is to be expected that the dot on the developed plate will be slightly elongated. The elongation will, however, not exceed 0.004 mm for an equatorial star and will be less at higher declinations, which is scarcely detectable and absolutely harmless to the measurement. Consequently a velocity-generating device with intermittent velocity changes was considered suitable. Furthermore it was realized that the most convenient digital velocity parameter for practical purposes might be $\sec\delta$. Accordingly the digital velocity-generating device was constructed as represented in Fig. 5.

The control desk for the instrument includes the $\sec\delta$-keyboard. A setting of e.g. $\sec\delta = 1.387$ is made by pressing four buttons 1-3-8-7, one in each vertical row. If a $\sec\delta$-value exceeds 9.999, e.g. $\sec\delta = 13.87$, the setting is done in the same way, except that the P-button also has to be pressed in this case. "P" represents polar stars, and the effect of the button is to shift the decimal point in the $\sec\delta$-value one place to the right. In this way 9000 different settings cover the range $1.000 \leq \sec\delta \leq 9.999$, and other 9000 settings cover the range $10.00 \leq \sec\delta \leq 99.99$. "L" represents lower culmination, and one of the effects of the L-button is to move the right

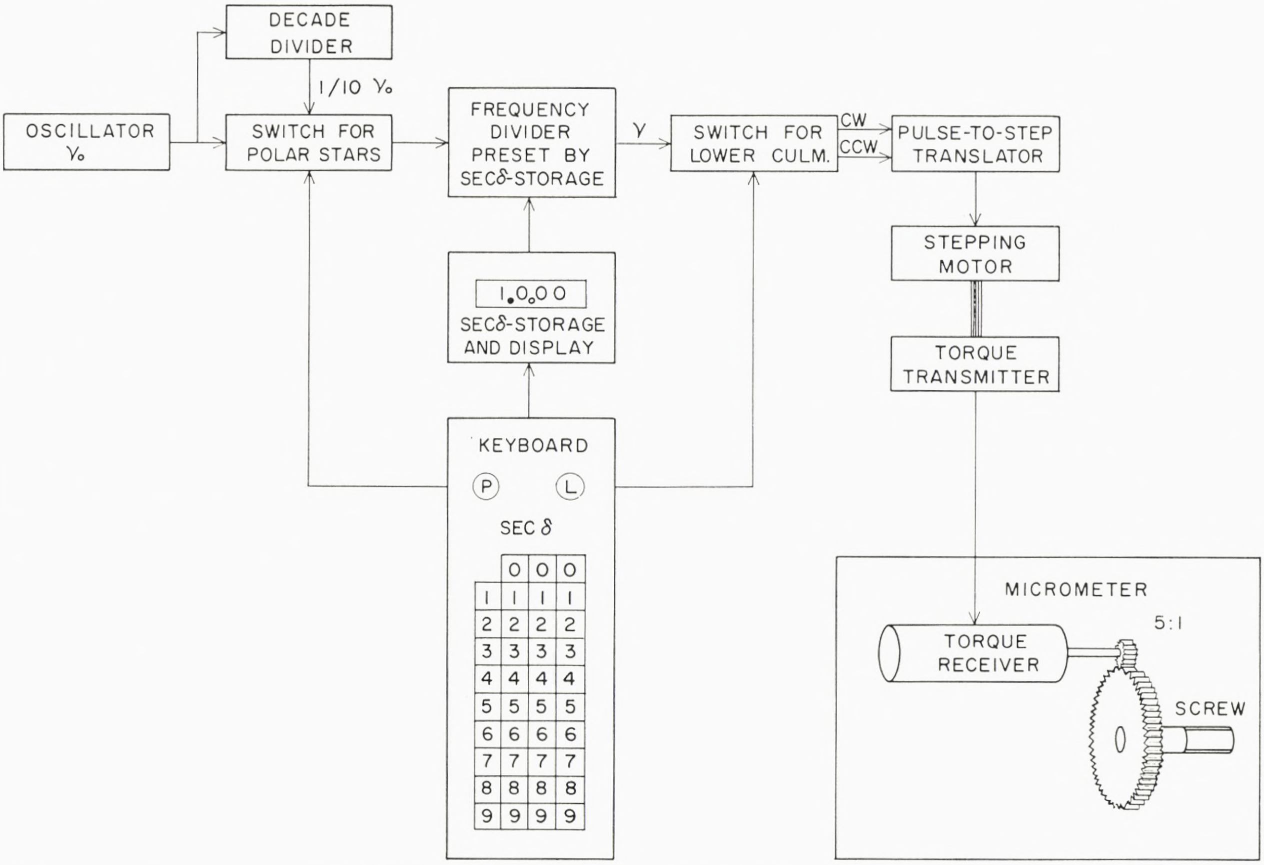

Fig. 5. Block diagram for the velocity-generating device.

ascension slide in the negative direction during the exposure, the direction of motion at upper culmination being defined as positive.

The sec δ-setting is stored in the sec δ-storage and made visible on the display with the correctly placed decimal point. The principal function of the sec δ-storage is to control the frequency divider. In addition, the sec δ-value in the storage is punched on paper tape together with the siderial time and other pieces of information necessary to secure the data needed for the reductions, and to achieve automatically the essential book-keeping. The tape punch and tape punch control are shown in the block diagram of Fig. 7.

The oscillator was designed by professor L. Stigmark, Lund, Sweden. The frequency is controlled by a low-priced quartz crystal without thermostat, which for this purpose suffices to secure an oscillator frequency of superior constancy. The oscillator frequency ν_0 has to be matched to the scale value of the instrument. Its present value is 387 680 c/sec. Any new scale value only makes necessary the replacement of the crystal by another one.

The oscillator signal with frequency ν_0 is transmitted to a decade divider, the output of which is $1/10\ \nu_0$. The divided signal as well as the oscillator signal is transmitted to a switch. For $1.000 \leq \sec\delta \leq 9.999$ the switch passes the signal with frequency ν_0 on to the frequency divider. However, for polar stars, $10.000 \leq \sec\delta \leq 99.99$, the P-button will cause the switch to select the signal with frequency $1/10\ \nu_0$ which is then passed to the frequency divider.

The frequency divider consists of four decade dividers, each of which is preset by the corresponding digit in the $\sec\delta$-storage. Here the contents of the storage are considered as an integral number. The division is performed by counting the input pulses from the preset value, for instance 1387, to zero. After this the four decades are reset, and simultaneously a pulse is delivered at the output of the frequency divider. The output frequency is called ν, and for all relevant values of $\sec\delta$ this frequency is given by the expression:

$$\nu = \frac{\nu_0}{1000 \cdot \sec\delta} \tag{2}$$

The frequency divider including the decade divider was designed by Mr. W. Hansen, A/S Regnecentralen, Århus. These units are all solid-state, and it is worthy to note that this essential part of the device has thus far proved to be very reliable.

The output signal from the frequency divider passes a second switch on its way to the pulse-to-step translator. For upper culmination the translator input for clockwise rotation is used, but when a lower culmination is observed, pressing the L-button causes the switch to select the input for counterclockwise rotation of the stepping motor.

The pulse-to-step converting unit is a SLO-SYN translator type ST-250 from the Superior Electric Company, USA, and the stepping motor is a Bifilar SLO-SYN Synchronous Motor type SS50-1009 from the same firm. The translator accepts the low-level signal pulses from the frequency divider at one of its input channels. The incoming signals are shaped in monostable circuits and transmitted to the logic elements, which provide the four-step sequence required to rotate the motor at 200 steps per revolution. The output from the logic elements feed two power flip-flops connected to the four motor windings, and in this way each pulse from the frequency divider is translated into a motor step equivalent to a motor shaft-rotation of 1.8 degrees, clockwise or counterclockwise according to the input channel selected. The rotational velocity of the stepping motor shaft is then given by the expression:

$$\frac{\nu_0}{200000 \cdot \sec\delta} \text{ revolutions/sec} \tag{3}$$

The stepping motor is connected with the rotor in a torque transmitter, which is electrically coupled to the torque receiver on the photographic micrometer. The torque transmitter and receiver performing a synchronous link between the stepping motor and a gear wheel in the micrometer are NIFEGON type 23TR6a, which are made by Svenska Ackumulator AB Jungner. The torsional elasticity of this electrical shaft is stated by the factory to be 0.11 degree per gcm. The torque needed to move the right ascension slide measures 80 gcm, corresponding to an angular deviation of 9 degrees, which again corresponds to a linear shift of 0.0025 mm of the right ascension slide. During the time of exposure this quantity will, no doubt, be sufficiently constant and thus the irregularity of the rate of motion is negligible.

The micrometer screw is driven by the torque receiver by means of a gear in the ratio 5:1. The micrometer screw pitch being 0.500 mm, the velocity of the right ascension slide is given by the expression:

$$V = \frac{\nu_0}{2000000 \cdot \sec \delta} \text{ mm/sec} \tag{4}$$

A comparison with equation (1) yields an expression for the matching of ν_0 to the scale value:

$$\nu_0 = 2 \cdot 10^6 \cdot V_e \tag{5}$$

where ν_0 is in c/sec and V_e in mm/sec. With the earlier mentioned scale value of 0.19384 we find $\nu_0 = 387\,680$ c/sec, which is in agreement with the oscillator frequency given above.

The uniformity of the rate of motion of the right ascension slide has been investigated. Series of flash marks were exposed on a moving plate at precise time intervals, which were controlled by the quartz clock. The marks were measured, and the deviations from a uniform rate were subsequently deduced by means of a least squares solution. The investigation covered the complete $\sec \delta$-range from 1.000 to 99.99, and the deviations were found to be slightly larger for higher $\sec \delta$-values. But the main results of the investigation was that the deviations, computed as a mean square error, were less than 0.001 mm for all $\sec \delta$-values.

5. Accurately Timed Flashes for Reference Marks

The reference marks on the photographic plate are produced by flash-light penetrating the two small openings in the graticule plate. As the photographic plate is moving across the graticule, the flashes must be short and accurately timed. Such requirements are, however, easily fulfilled by modern technique.

The principal timekeeper at the Observatory is a mean-time quartz clock type CAQ from Rohde and Schwarz, Munich, West Germany. It is a small, all solid-state, quartz clock, which is normally connected to the mains, but it also has a built-in battery storage sufficient for 15 hours operation. The day-to-day stability was found to be better than two milliseconds per day.

The quartz clock is connected with a sidereal time converter. The input is a

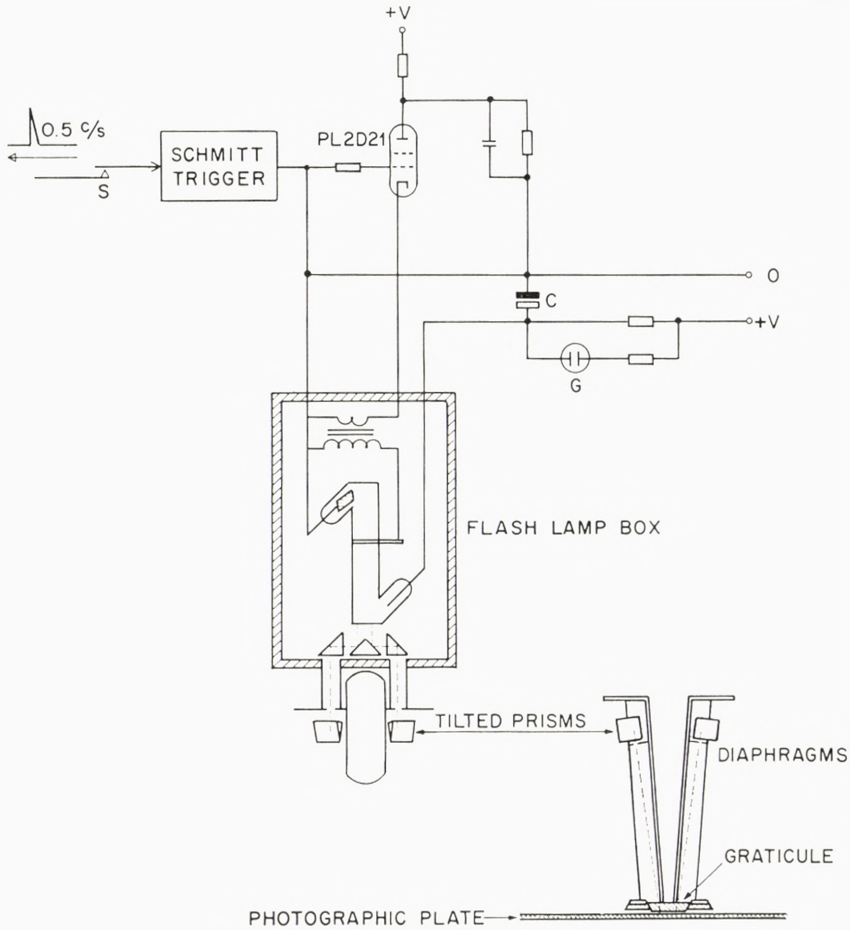

Fig. 6. Flash arrangement.

sine wave signal of 1 kc/sec mean solar time, which is converted to 1 kc/sec sidereal time by means of a motor driven goniometer. In addition to the goniometer device the sidereal time converter contains several frequency divider units with various outputs. One of the outputs is a 0.5 c/sec signal composed of accurate positive-negatively alternating pulses, and the whole seconds by the sidereal clock are, by definition, coinciding with the sharp front edges of these pulses. Comparison between the quartz clock and the sidereal time converter is performed by means of a time signal oscillograph, and the clock correction is determined in the same way by comparing the quartz clock with radio time signals.

Thus the front edges of the second pulses from the sidereal time converter are highly suitable for accurate timing. The diagram of Fig. 6 shows the principle for firing the flash, the flash lamp mounting and the path of rays towards the photographic plate. The function of the switch *S*, which is controlled by the master control unit, is to allow the passage of the second pulses into a Schmitt trigger circuit at selected times. The trigger is only released by positive pulses, corresponding to accurately timed, even sidereal seconds. The trigger is connected with a thyratron circuit, which supplies the ignition coil for the flash lamp with power. Consequently, when a positive second pulse is transferred to the trigger, the photoflash will fire and discharge the capacitor *C*. As a check a neon lamp *G* on the control panel is lit approximately one second after the capacitor is discharged.

The flash lamp is a lens-end type which radiates the bulk of the light in a cone pointing towards two aluminized sides in a right-angled prism as seen in Fig. 6. Each of the two light beams from the first prism are reflected by two other prisms, one of which is tilted some seven degrees to the optical axis of the instrument so as to throw the light through the circular diaphragms of 1.0 mm diameter close to the prisms towards the openings in the graticule on to the photographic plate. Owing to the size of the diaphragms, which are located 36 mm from the graticule, the light beams through the graticule openings are quite narrow and therefore produce sharp images on the photographic plate.

6. Timing and Control of Observations

The performance of an observation is necessarily of a complex nature. Different functions have to be performed in a specified order and be more or less accurately timed. It was, therefore, considered desirable to attain a high degree of automation. Furthermore, the book-keeping procedure would be simplified if the observations were accompanied by automatic data recording, because one night may yield a few hundred observations. The control units relating to timing and control of observations and data recording are shown in the block diagram of Fig. 7, while the actual timing of observations is displayed in the schematic diagram given in Table 1.

A square wave sidereal time signal of 0.5 c/sec is used to control the sidereal slave clocks at the observatory. One of the slave clocks is the observation timer at the transit circle control panel, which shows hours, minutes and even seconds, in as much as it advances every two seconds. The observation timer is a digital clock consisting of rotary switches with six parallel sets of contacts. One set of contacts is used for display for each digit, another set for recording the time on punch tape, and for the last three digits a third set of contacts form part of a starting device. Experience has clearly demonstrated that it is more convenient to preset the starting time than to activate a pushbutton starter at the proper moment for each observation. By means of a key-board on the control desk, the observer sets the last digit to the minute and the second corresponding to the approximate right ascension of the star. The starting time can be set up to ten minutes before the observation begins; usually this presetting is done during the previous observation. When the observation timer for the three relevant digits coincides with the preset "right ascension", the starting device in the timer automatically starts the observation.

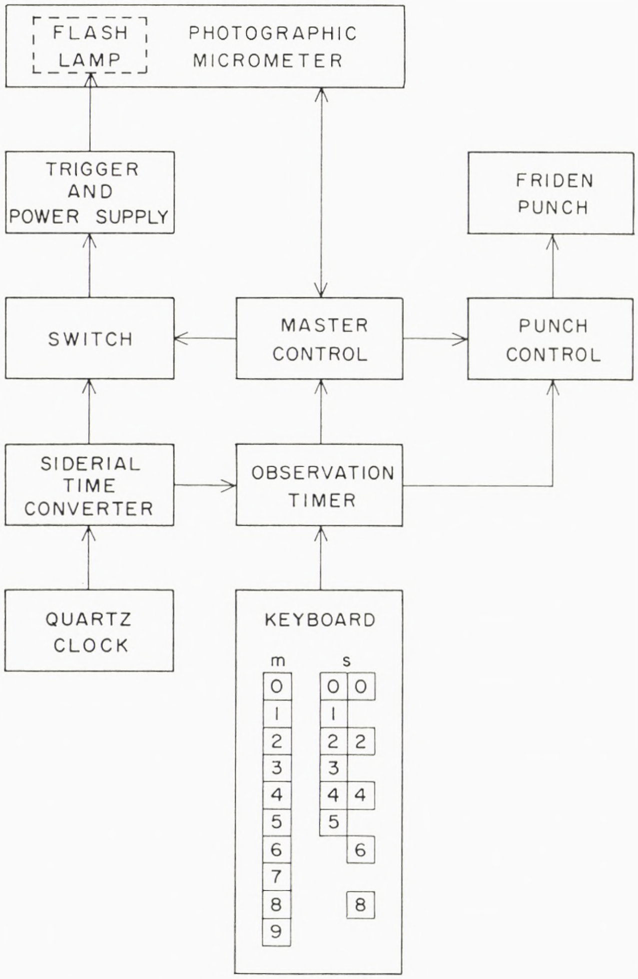

Fig. 7. Block diagram of timing and control units.

Generally the exposure time is 20 seconds, and as it is desirable to expose symmetrically with respect to the meridian, the exposure has to begin 10 seconds before transit. As, furthermore, each exposure must be prepared in various ways, the observation is started 18 seconds (for extended exposure 28 seconds) before the transit of the star concerned. The observation timer is, on the other hand, set 18 seconds ahead of the sidereal clock, and therefore the digits to be set on the right ascension keyboard agree with the approximate right ascension of the star. The recorded time will also represent the approximate right ascension of the star as the punching of the time according to the observation timer is made immediately after the start of the observation.

The observations are timed according to the schedules in Table 1, to which

TABLE 1. Timing of observations.

Time scale sec.	Upper culmination $1.000 \leq \sec\delta \leq 9.999$	Upper culmination $10.000 \leq \sec\delta \leq 99.99$	Lower culmination $1.000 \leq \sec\delta \leq 9.999$	Lower culmination $10.00 \leq \sec\delta \leq 99.99$	Extended exp. Time scale sec.
0	start of observation	start of observation	start of observation	start of observation	0
0–4			slide advance: $V = +5V_e$	slide advance: $V = +5V_e$	0–4
1–3	data recording	data recording	data recording	data recording	1–3
4	start of slide: $V = +V_e\cos\delta$		start of slide: $V = -V_e\cos\delta$		4
4–6		slide motion: $V = +10V_e\cos\delta$		slide motion: $V = -10V_e\cos\delta$	4–6
6		start of slide: $V = +V_e\cos\delta$		start of slide: $V = -V_e\cos\delta$	6
8.000		first reference mark flash		first reference mark flash	8.000
$8 + a$	start of exposure	start of exposure	start of exposure	start of exposure	$8 + a$
14.000	first reference mark flash		first reference mark flash		24.000
$18 + (a + b)/2$	mid-exposure	mid-exposure	mid-exposure	mid-exposure	$28 + (a + b)/2$
22.000	second reference mark flash		second reference mark flash		32.000
28.000		second reference mark flash		second reference mark flash	48.000
$28 + b$	stop of exposure	stop of exposure	stop of exposure	stop of exposure	$48 + b$
28.20	stop of slide	stop of slide	stop of slide	stop of slide	48.20
28.20–30			slide advance: $V = +5V_e$	slide advance: $V = +5V_e$	48.20–50
8–24	declination circle recording	declination circle recording	declination circle recording	declination circle recording	8–24
24–40	return of circle micrometers	return of circle micrometers	return of circle micrometers	return of circle micrometers	24–40

only a few comments are necessary. The motion of the right ascension slide is measured positive to the east and negative to the west. At lower culmination the slide moves in the negative direction during the exposure, and therefore it has to be advanced before and after such exposure in order to avoid mixing with adjacent observations. For high declination stars the right ascension slide moves slowly, and in order to make sure that the slide has started moving when the exposure begins, it is for $\sec\delta \geq 10.00$ run with ten times the proper speed for two seconds. For $\sec\delta \geq 10.00$ the flashes are more widely spaced in time in order to avoid merging the reference marks on the plate. It is possible to extend the exposure time to 40 seconds in order to reach the fainter stars, in which case the right-hand time scale in Table 1 is valid.

The beginning and end of an exposure takes place with a certain time lag, denoted by a and b, respectively, due to the time constants for rotary switches, relays and the shutter coil. The quantities a and b have both been found equal to $0\overset{s}{.}14$ from measurements, and thus the time for mid-exposure differs from the mean of the two moments for flashing by the same amount. This deviation will introduce an error, if the rate of the photographic plate is not in exact correspondence with the rate of the star. The right ascension slide is, however, adjusted to the speed of the star image within a limit of 0.1 per cent, and the error in right ascension due to the time lag of the shutter is, therefore, less than $0\overset{s}{.}00014$, which is insignificant.

7. Photographic Registration of the Circle

Seven cameras for photographic registration of the circle were constructed in the observatory workshop shortly after the erection of the instrument. Six of these cameras were used for the accurate registration of the circle setting, while one camera was for the low-power microscope. This last one is still in use in connection with the photoelectric recording now applied. Exposures of 35 mm films were made by switching on the light for five seconds in the lamp houses at the front end of the circle microscopes, and therefore no shutters were needed in the cameras. Exposure and film feed were controlled by the photographic micrometer master control unit.

The measuring of the declination circle photographs was effected by means of an automatic photoelectric measuring engine. Its mechanical part was built in the observatory workshop, mainly as a copy of the U.S. Naval Observatory measuring engine constructed by C. B. Watts(1), whereas a new electrical control system was designed and constructed by P. Naur(2). The principal features of the original design were utilized throughout, although several alterations were made since the engine began regular service. The mechanical part was simplified by removing the lower screw, so that a single micrometer screw was used for pointing on the reference line and the division lines. A substantial advantage was derived by measuring in both directions on the films, so that one division line on each side of the reference line was measured. Thereby a first check was possible while measuring, the accuracy of the measurements was increased, and the scale value of each microscope was automatically determined every night. The system for photographic recording of the measures was replaced by a digitizer attached to the micrometer screw. Most of the improvements were carried out by Mr. H. Schnedler Nielsen, who designed and constructed the new control system in 1961. The digital recording system was developed by him in co-operation with the writer.

8. Photoelectric Recording of the Circle

The first experiment with a direct and automatic method for recording the circle was made in October 1964. A slit-plate with a photomultiplier tube behind it was attached to one of the circle microscopes, and a recording equipment with digital output was connected to the photomultiplier, while a stepping motor was coupled through a gear box to the slow-motion screw of the instrument. In this way it became possible to scan the circle and thereby to record profiles against a continuum, corresponding to the dark division lines against the bright background of the circle. Repeated measurings of a section of the circle immediately showed that photoelectric recording might yield at least the same accuracy as the photographic method. It was, therefore, decided to begin the design and construction of six photoelectric micrometers and a matching data logging system. As the new equipment had to be adopted to the existing circle, it was found preferable to retain also the existing microscopes, in order to reduce the cost and time in the conversion to the new system.

The micrometers were constructed in the observatory workshop. The photograph of Fig. 8 shows the west pier with the assembled micrometers mounted on the circle microscopes. The operational procedure is shown by the cross-sectional view of a micrometer, Fig. 9.

The micrometer box is fastened to the microscope tube by means of a short tube adaptor. At the end of the tube adaptor inside the micrometer box is attached a small glass plate *R*, which serves as a fixed reticle. On its surface away from the microscope objective, the reticle has the pattern shown in Fig. 9. The four lines are 7.0 mm in length and 0.15 mm wide, and the distances between the lines are 3.5 mm, which corresponds to 5 minutes of arc on the circle. A second glass plate, the slit plate, is placed parallel to the reticle plate at a distance of 0.2 mm. The pattern on the side towards the reticle plate is also shown in Fig. 9. The length and width of the slit are 5.0 mm × 0.15 mm, and the black line 7.0 mm to the right of the slit is of the same size.

The slit plate and the photomultiplier tube are mounted on a carriage, which is guided by a cylindrical way and held in position by a set of ball bearings. The movement of the carriage along the guide is controlled by a micrometer screw coupled through a gear box to a synchronous motor designed for reversal duties. The rate of the motor is 50 rev/sec, and with the gear ratio of 50:1 and a screw pitch of 0.500 mm the carriage moves with a speed of 0.500 mm/sec, which corresponds to approximately 43 seconds of arc on the circle per second.

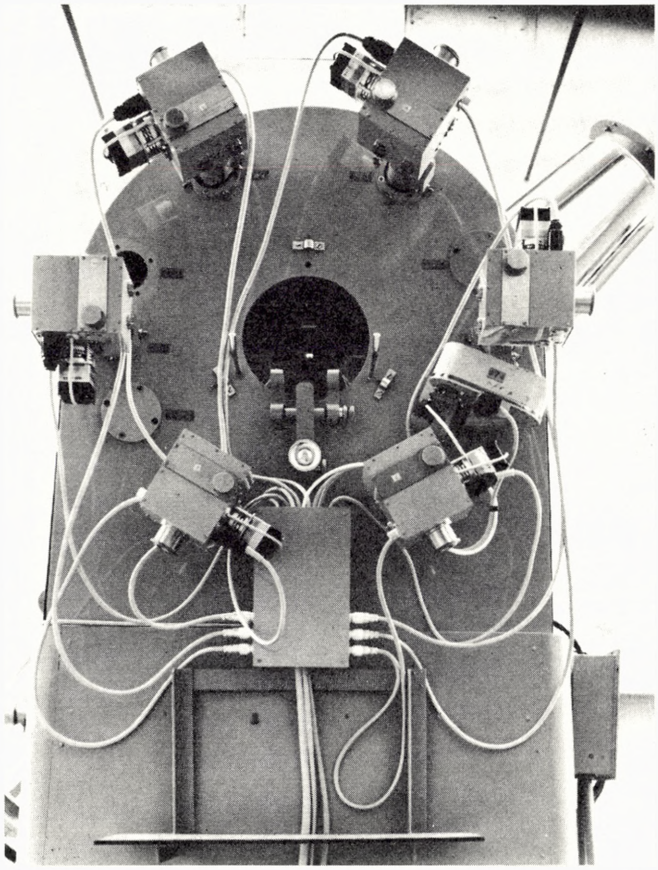

Fig. 8. The west pier with six photoelectric micrometers mounted on the microscopes 60 degrees apart. The photographic rough reader is also visible in the picture.

The photomultiplier tubes, which are of the 931A type from Hamamatsu TV Co., Japan, were selected on the basis of similarity in sensitivity. They are effectively protected from false light by cylindrical metal cases with openings towards the slits only. An optical system for visual inspection was built into the micrometers for adjusting purposes. For the same purpose the right-hand third of the slit plate was made transparent and provided with a reversed image of the slit at a distance corresponding to 10 minutes of arc from the true slit, and the reticle plate was provided with the line R_0. Through the eye piece these lines are visible together with the division lines of the circle, and thereby all necessary adjustments can be made.

Two limit switches (not shown in the drawing) are mounted in each micrometer in order to delimit the range to be scanned. In present practice, 11–12 minutes of arc are scanned in each measuring, which according to the above mentioned rate is equivalent to a recording time of 16 sec. A measuring always includes the three reference lines R_1, R_2 and R_3 of the reticle and two division lines of the circle spaced

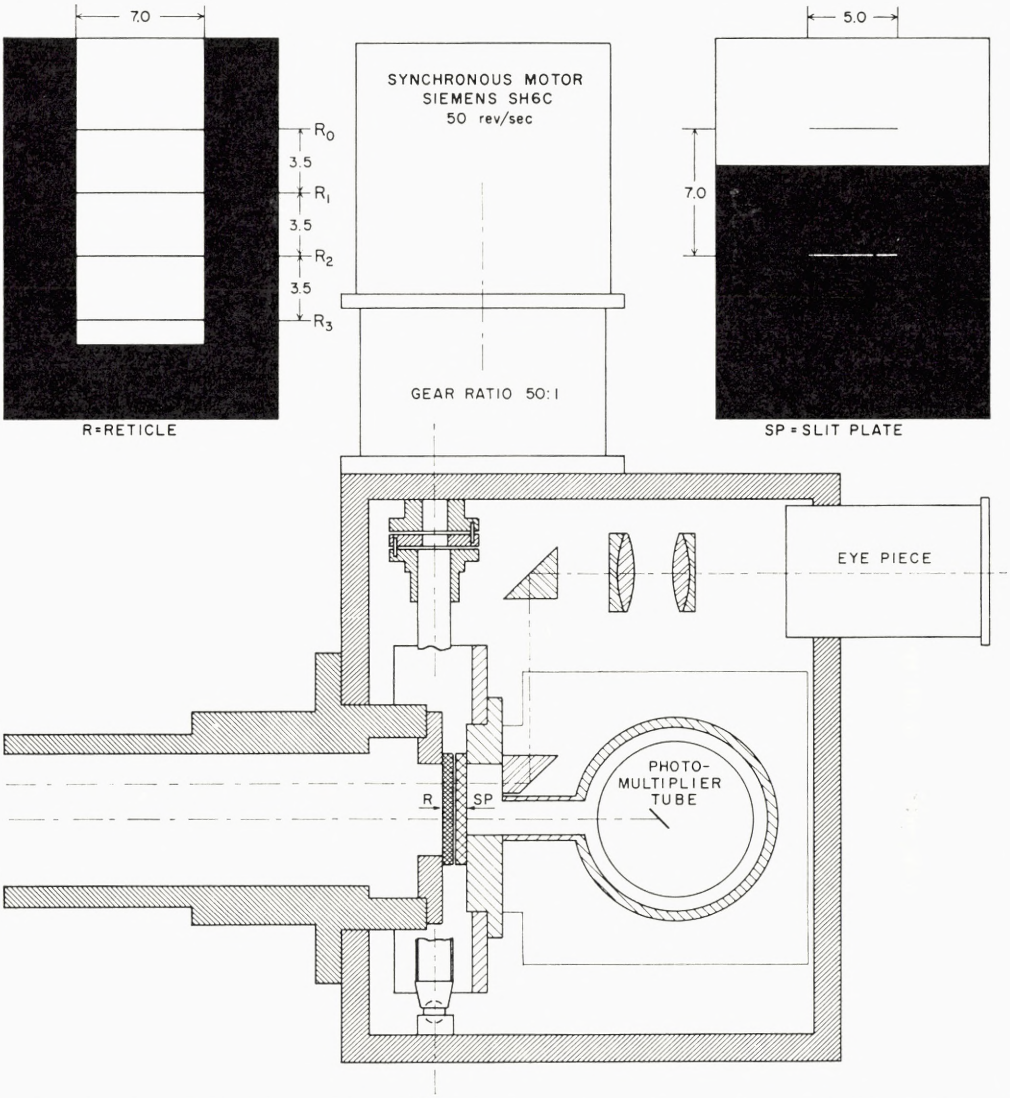

Fig. 9. Drawing showing the photoelectric micrometer for circle reading. At the upper left and right corners are shown in a larger scale the reticle and the slit plate.

between the reference lines. Circle settings resulting in blending of reference lines and division lines are forbidden. Measuring can be started either manually or by a command from the photographic micrometer master control unit, as indicated in Fig. 11. The carriages are set in motion by the motors which are all started at the same time by means of the control exerted by the motor control unit. As a uniform speed of the carriages is critically important, a 50 c/sec signal from the quartz clock is utilized as frequency standard for the driving current feeding the synchronous motors.

The data logging system was constructed according to our specifications by A/S Regnecentralen, Copenhagen. The equipment, which consists of a set of all solid-state modules, is shown at the bottom of the middle rack in Fig. 2. The working principle of the system is illustrated by means of the block diagram shown in Fig. 11.

The data logging system has six analog input channels, each of which is provided with an amplifier, coupled as a current to voltage amplifier. Fig. 10 illustrates the principle of the amplifier, and the transfer characteristics are given in Table 2. The amplification can be varied in steps from 2.5 $V/\mu A$ to 5 $V/\mu A$, and simultaneously

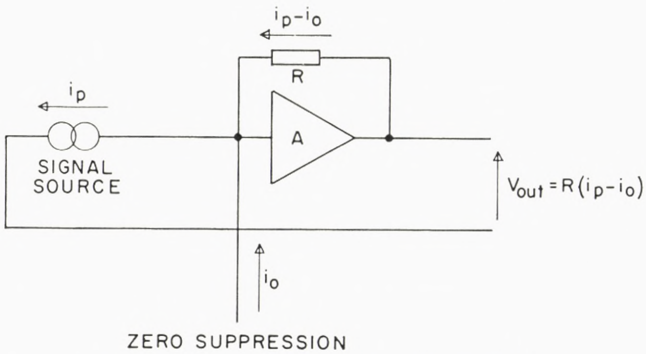

Fig. 10. Principle of current to voltage amplifier.

the zero points of all the input channels are suppressed to the fixed amounts shown in Table 2. Zero suppression and simultaneous selection of amplification has been introduced in order to make the best possible use of the capacity of the paper tape; in this way the highest possible accuracy is attained.

The outputs of the amplifiers are connected to the analog-digital converter through a multiplexer, which is composed of a series of transistor contacts connecting the analog input channels to the converter, one by one. The analog-digital converter was designed to convert analog voltage signals in the range 0 to +5 volts into a 7-bit binary number. A 7-bit binary number can assume 128 different values. The value 0 corresponds to 0 volt and the value 127 to +5 volts. The 7-bit numbers representing the measuring results are recorded on 8-track punched paper tape, one number on each character. The maximum value 127 is, however, reserved for an administrative purpose, and therefore a measuring result giving the value 127 is punched as 126. The conversion principle of the analog-digital converter is that of successive approximation, and the conversion moments are controlled by the master control unit.

The master control unit is a program unit for controlling the acquisition of analog and digital data and for controlling the recording of the digital data. The unit includes a scan control for the analog multiplexer, a digital multiplexer and its control circuits, a buffer system for one 7-bit number, and a synchronization and timing unit. The object of the digital scan control and multiplexer is to ensure that the digital channels are scanned in the order *A*-*B*-*C*. The digital channels *A* and *C*, each of 7 bits, are used for semipermanent digital data; channel *B* is connected to the analog-digital converter for recording of the 7-bit binary number representing the analog input signal. Channel *A* is connected to a set of toggle switches, by means of which a 7-bit binary number can be set up. On channel *C* the number 127 is set up permanently. The character with the number 127 acts as a stop code in the later handling of the data.

The analog scan control exerts control of the analog multiplexer in such a way that a scanning sequence is always started with channel 1 and subsequent channels in numerical order, and channel *B* of the digital multiplexer always remains selected at least as long as one complete scanning sequence of the six analog input channels is performed. Normally we are, however, concerned with consecutive scannings of

TABLE 2.

Zero suppression	μA	0.00	0.25	0.50	0.75	1.00
Amplification	$V/\mu A$	2.50	2.86	3.33	4.00	5.00

the six analog input channels, as the scanning process is continously repeated until a stop order is given. The effect of the stop order is that after completion of the actual scanning sequence of the analog multiplexer, the channel *C* of the digital multiplexer is selected, and a moment later the recording process stops.

The 7-bit binary numbers are recorded together with a parity bit, generated by the punch control unit, in an 8-track paper tape by means of a Teletype tape punch. The rate of the measurements through data channel *B* is 100 per second, and accordingly the recording rate is 100 characters per second. The timing of the conversions in the analog-digital converter must be strictly accurate, because the time scale for the measurements is equivalent to a distance scale in each micrometer, and our aim is the determination of distances between reference lines and division lines. Therefore the time control is exerted by the quartz clock, which through the synchronization and timing circuits of the master control unit starts a conversion every ten milliseconds. The quartz clock thus controls the time scale of the output tape in the same way as it controls the micrometer motors, and thereby an adequate synchronization is established.

The record of a circle setting comprises approximately 1600 characters and covers 4 metres of paper tape. The first character is the number from data channel *A*, which is set up by means of the toggle switches. This is followed by a multiple of six characters, all of which are 7-bit binary numbers from data channel *B*. The character closing the record is the stop code from data channel *C*.

Such extensive data collection, naturally requires processing with an electronic computer. The GIER computer at the Copenhagen Observatory was utilized for all data processing in connection with the transit circle. First the data of a record are sorted according to the micrometer from which they originated, then the data from

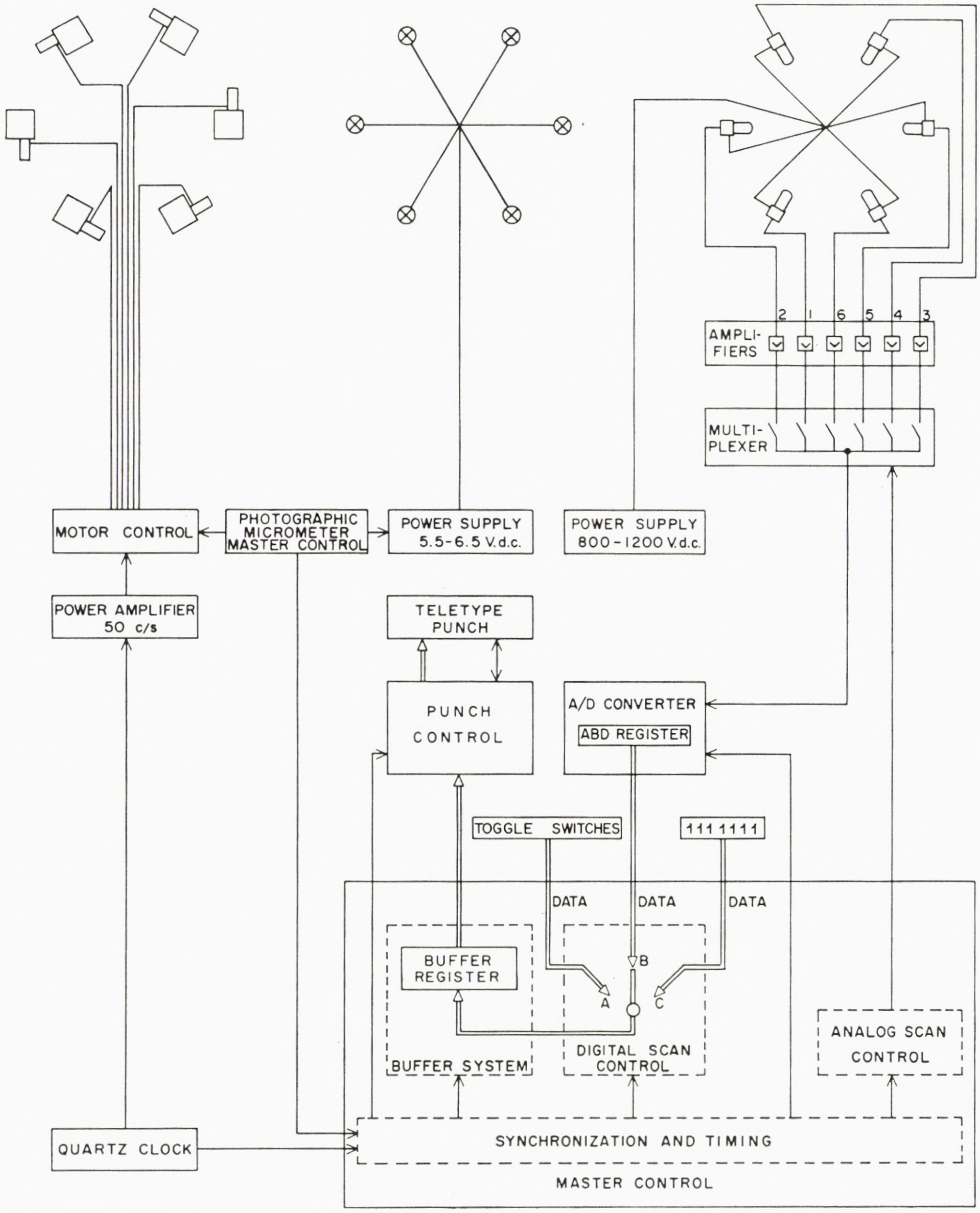

Fig. 11. Block diagram for the photoelectric recording system. The various functions are shown separately in the upper part of the diagram, to the left are the six micrometers with motors, in the middle six lamps, and the six photomultiplier tubes with connection for high voltage power supply and the output signals are seen to the right.

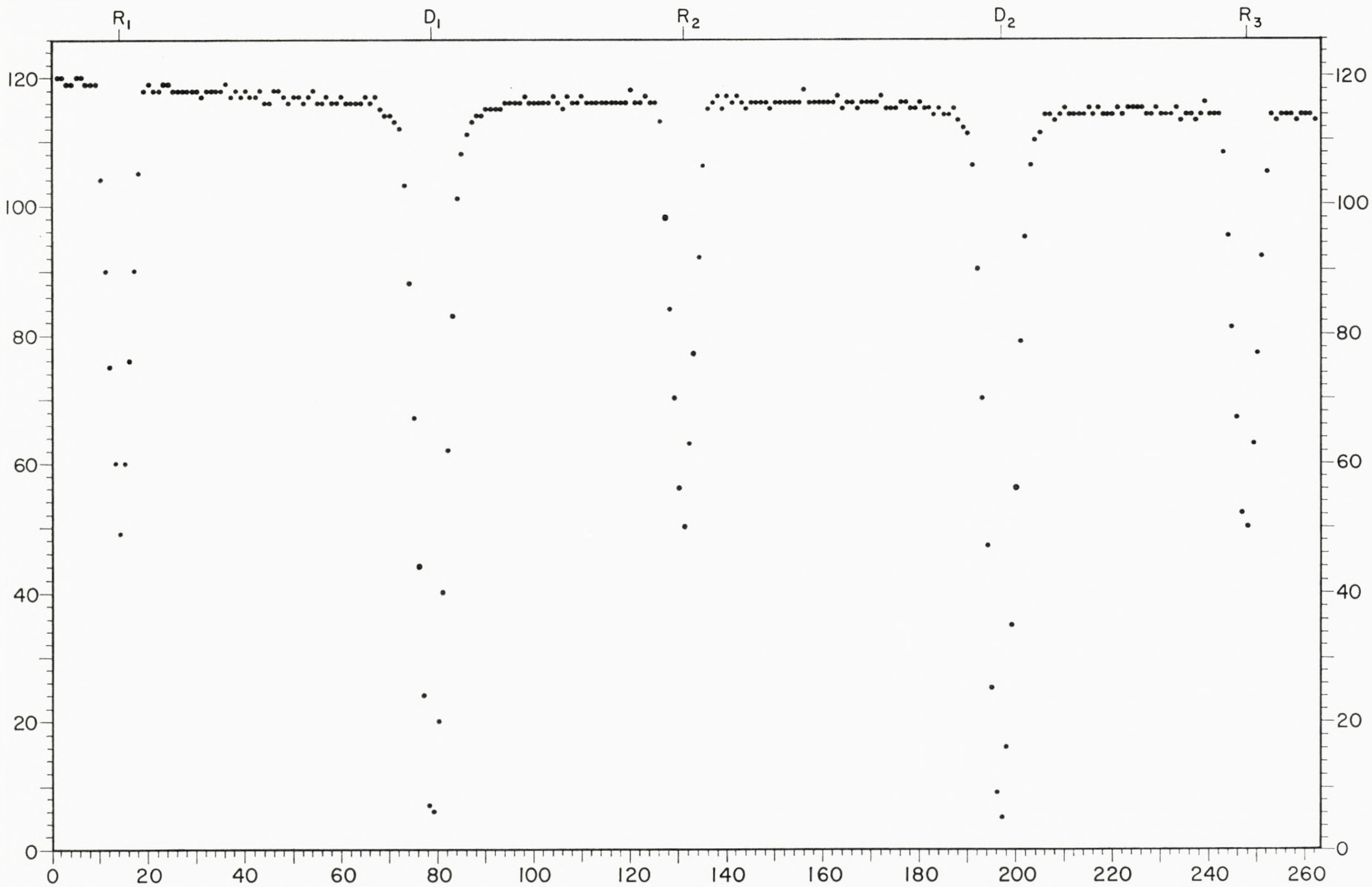

Fig. 12. Plot of data output from one of the six micrometers. The high level continuum is intersected by the reference line profiles R_1, R_2 and R_3 and the division line profiles D_1 and D_2. The depths of the reference line profiles are lower than those of the division lines; new reticle plates with heavier lines are, however, expected for delivery early in 1967.

each micrometer are handled separately. In Fig. 12 are plotted the data recorded from one micrometer. The abscissa is the number n of the measurement concerned, and the ordinate is the digital output y_n of the measurement. The outputs comprise a continuum intersected by five profiles, of which R_1, R_2 and R_3 are due to the reference lines of the reticle and D_1 and D_2 arise from the division lines of the circle.

The reduction procedure is quite simple. The computer seeks the profiles one after the other and computes their abscissa value in the sense of Fig. 12 by means of the expression:

$$N = n + \frac{2}{3} \cdot \frac{(y_{n-4} + y_{n-3} + y_{n-2}) - (y_{n+2} + y_{n+3} + y_{n+4})}{(y_{n-4} - y_{n-2}) - (y_{n+2} - y_{n+4})} \tag{6}$$

Here n is the number identifying the lowest of all y-values in the profile, or in case of two equal values at the bottom of the profile, by convention, always the last of the two. The last term in the expression is a decimal fraction of the abscissa unit, and the computations are performed to three decimal places. The value of N, according to the computation above, is identical with the abscissa of the intersection between the two straight lines, which are assigned to the three points: $(n-1, y_{n-4})$, (n, y_{n-3}), $(n+1, y_{n-2})$ and the three points: $(n-1, y_{n+2})$, (n, y_{n+3}), $(n+1, y_{n+4})$ by means of two least squares solutions. The reasons adduced for this procedure are that the two sides of the profiles are found to be rectilinear within the ranges, from which measurements are used in the reduction.

The accuracy in the determination of N has been investigated, and a mean error of 0.016 abscissa units was found. This corresponds to a linear distance of 0.00006 mm or an angular distance of 0.″04 on the circle.

Every record yields for each micrometer a determination of the scale value, as the distance between D_1 and D_2 is put equal to 300″. An approximate value is 2.″56 per abscissa unit, but in practice the scale value is determined as the average of many measurements and with an accuracy of about 0.″0001 per abscissa unit.

The position of the circle is determined from a record containing six scannings similar to the one shown in Fig. 12, and the determination thus utilizes six circle diameters. The diameter corrections applied so far were determined in 1962 by the photographic method, but a new determination was made in January 1967 by means of the photoelectric method. The measurements for all 2160 diameter corrections were carried out in about 15 hours and the complete reduction in 20 hours. It is, therefore, intended to repeat the determination in a few months in order to investigate the stability of the glass circle. The accuracy of the new determination was high, the mean error of a single diameter correction being 0.″026. When these diameter corrections are applied the circle position may be determined with a mean error of about 0.″03.

In order to turn the accuracy of the method to the best possible account the division lines and their illumination deserve closer examination. The measured position of a division line depends on the illumination, and this dependence was found critical, as long as the original lamp houses were used. Dark and light areas were visible on the surface of the paint-stripes, but the pattern could never be accurately reproduced after a lamp house had been touched. New lamp houses were, therefore, designed and has recently been installed. First the application of light guides from a single central lamp to the six lamp houses made it easier to secure a well defined light source in each lamp house and it reduced, furthermore, the risk of heat-transfer from the lamps to the microscopes or the circle. Secondly, the introduction of a narrow tube, which extends close to the circle and thereby affords shade on the front side of the paint-stripes, removes their pattern and make the division lines quite dark against the light background of the white-painted sheet behind the glass disc.

9. Basic Principles for the Reduction of Observations

The axis of rotation intersects the celestial sphere in the instrumental east point E' and the instrumental west point W', which are close to the true east point E and west point W. The deviations are described by the Besselian constants:

$$m = \alpha(W') - \alpha(W) = 6^h - t(W') \tag{7}$$

$$n = \delta(W') \tag{8}$$

The instrumental meridian plane, which contains the optical centre of the objective, is perpendicular to the axis of rotation. The intersection of this plane with the celestial sphere defines the instrumental meridian, which is, therefore, a great circle with E' and W' as its poles.

An observation with the micrometer results in a number of marks on the developed plate. The reference marks define the position of the photographic plate at the moment of flashing. The reference marks on the plate or these marks projected by the instrument on the celestial sphere at the flashing moment are denoted by R_1 and R_2 as shown in Fig. 13. The point bisecting the line segment R_1R_2 on the plate or the corresponding point in the sky bisecting the great circle arc R_1R_2 is the micrometer centre R. The observed star appears as a circular dot on the plate. In Fig. 13, this dot S is seen projected on the celestial sphere at the flashing moment. The position of S as related to the system of reference marks is measured on the plate and expressed by the quantities Δc and Δz in the CZ-coordinate system. The location of this coordinate system on the plate is determined by the reference marks together with the collimation error c and the micrometer position angle $V = W'RR_1 - 90°$.

The observed right ascension of a star α is determined from the relation, $\alpha = \theta + \tau$, where θ is the apparent sidereal time, and the quantity τ is derived from the spherical triangle $W'SP$:

$$-\sin(c + \Delta c) = \sin\delta \cdot \sin n - \cos\delta \cdot \cos n \cdot \sin(\tau - m) \tag{9}$$

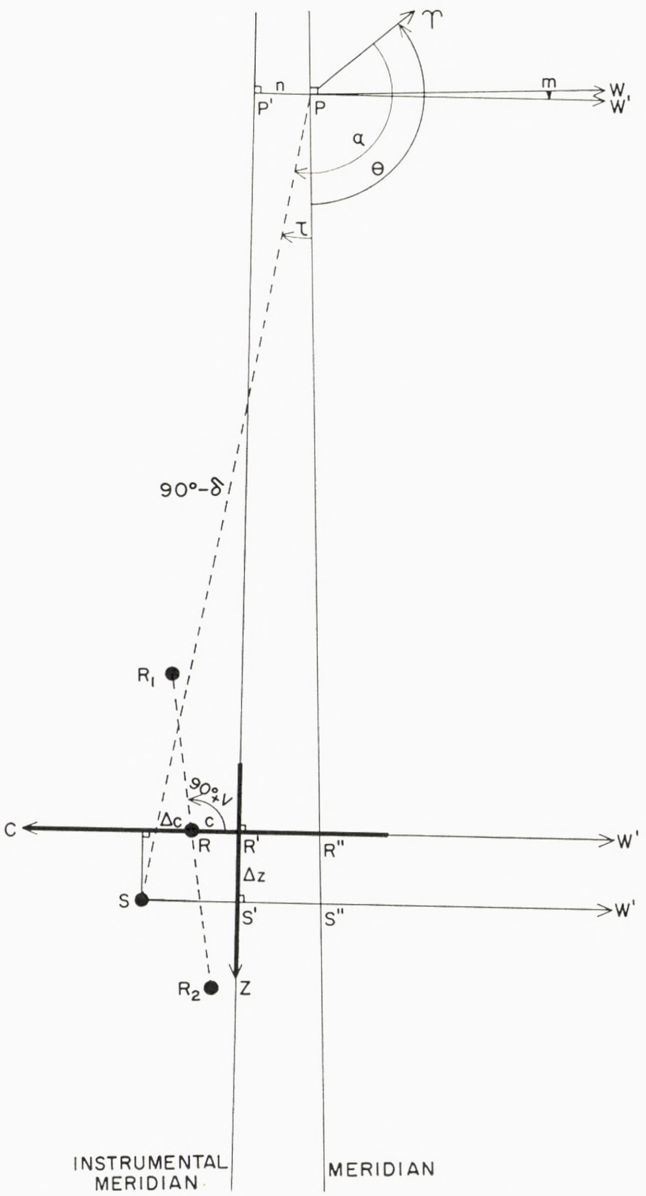

Fig. 13. The CZ-coordinate system projected on the celestial sphere.

The instrumental constants m, n and c are all small quantities; this also applies to Δc as the time of mid-exposure is at most a few seconds of time off the apparent transit time. For all stars, except those very close to the pole, $(\tau - m)$ is therefore a small angle at upper culmination and close to $180°$ at lower culmination. Consequently, the following expressions for right ascension hold with a good approximation:

$$\text{upper culmination:} \quad \alpha = \theta + m + n \cdot \mathrm{tg}\,\delta + (c + \Delta c) \cdot \sec \delta \tag{10}$$

$$\text{lower culmination:} \quad \alpha = \theta + m - n \cdot \mathrm{tg}\,\delta - (c + \Delta c) \cdot \sec \delta + 12^h \tag{11}$$

The declination is determined by the expression:

$$\sin\,(90° - \delta) = \sin\,(P'R' + \Delta z) \cdot \cos\,(c + \Delta c) \cdot \sec\,(\tau - m) \tag{12}$$

where $P'R'$ is measured from P' in the direction of the instrumental zenith Z' from 0° to 360°. In this way the expression is valid for both upper and lower culminations. The quantity $(P'R' + \Delta z)$ is assumed to be corrected for refraction. The numerical value of the factor $\cos(c + \Delta c) \cdot \sec(\tau - m)$ is quite close to unity, and the factor can be omitted in most cases.

10. Observational Results

The construction of the photographic micrometer and electronic control device was completed in the summer 1964, and the transit circle has been in use since then except for minor interruptions. The observational total in the period from August 1964 until the end of January 1967 was 10 000 observations. The measuring of the plates and the reduction of these observations are expected to be finished within a few months.

The observations are strictly differential, and the aim is to tie the program stars as close as possible to the FK4 fundamental system. The program stars are observed in declination zones not exceeding 30°, and final corrections to the program star positions are derived from FK4 stars in and near the zone. The collimation error is determined by means of the collimators every two hours during the night, whereas the Besselian constants m and n are determined two or three times per hour from pairs of FK4 stars, one clock star and one pole star at upper or lower culmination. Smoothed values of m, n and c are used in the reductions.

Night-to-night comparisons for determining the internal errors of a single observation in right ascension and declination have given the results shown in Table 3. The stars have been divided in two groups, the first one contains stars brighter than $m_v = 8.5$. The second group covers the range $8.5 \leq m_v \leq 10.0$. For these fainter stars the accidental errors are somewhat larger than for the brighter stars. The increase in error can partly be explained by the fact that in the autumn 1965, plates of poor quality reduced the quality of the images of the fainter stars.

The limiting magnitude of the instrument and method discussed is well below $10\overset{m}{.}0$. The internal errors for the faintest stars cannot yet be stated with certainty, but observations thus far indicate that the errors for stars close to $11\overset{m}{.}0$ are of the same size as for the second group in Table 3, when an exposure time of 40 seconds is used on Kodak 0a-D plates.

Table 3. Internal mean errors of one observation.

	$m_v < 8.5$	$8.5 \leq m_v \leq 10.0$
Number of observations	1367	320
$\varepsilon_\alpha \cdot \cos\delta$	$0\overset{s}{.}013$	$0\overset{s}{.}015$
Number of observations	913	226
ε_δ	$0\overset{''}{.}20$	$0\overset{''}{.}24$

Acknowledgments

The construction of the photographic micrometer with control units and the photoelectric recording system was made possible by grants from the Carlsberg Foundation.

It is a pleasure for me to thank the Director of the Copenhagen and Brorfelde observatories, Professor A. Reiz, for his constant and stimulating interest in this work and for reading the manuscript. I also want to express my gratitude to Professor Bengt Strömgren for valuable discussions concerning my work.

I am further indebted to the head of the observatory workshop, Mr. P. Bechmann, who constructed the photographic micrometer and the micrometers for photoelectric recording of the circle. Assistance from outside the observatory came principally from the technical staff of A/S Regnecentralen. Mr. W. Hansen designed the frequency divider for the velocity-generating device, and Mr. M. Strange was the principal designer of the data logging system for the photoelectric recording. The author greatly appreciates their collaboration in producing this excellent equipment.

I have been helped greatly by Mr. Poul Jensen and Mr. Torben Knudsen, assistants at the Brorfelde observatory. I am truly grateful to them for their outstanding cooperation.

References

(1) C. B. Watts, 1950. Publ. of the U. S. Naval Obs. Vol. 16 part 2.
(2) P. Naur, 1958. Publ. og mindre Medd. fra Københavns Obs. Nr. 174.

Indleveret til Selskabet den 14. februar 1967.
Færdig fra trykkeriet den 27. juni 1967.

Det Kongelige Danske Videnskabernes Selskab

Matematisk-fysiske Skrifter

Mat. Fys. Skr. Dan. Vid. Selsk.

Bind **1** (kr. 141.–)

		kr. ø.
1.	BRODERSEN, SVEND, and LANGSETH, A.: The Infrared Spectra of Benzene, sym-Benzene-d_3, and Benzene-d_6. 1956	14.–
2.	NÖRLUND, N. E.: Sur les fonctions hypergéométriques d'ordre supérieur. 1956	15.–
3.	FRÖMAN, PER OLOF: Alpha Decay of Deformed Nuclei. 1957	20.–
4.	BRODERSEN, SVEND: A Simplified Procedure for Calculating the Complete Harmonic Potential Function of a Molecule from the Vibrational Frequencies. 1957	10.–
5.	BRODERSEN, SVEND, and LANGSETH, A.: A Complete Rule for the Vibrational Frequencies of Certain Isotopic Molecules. 1958	6.–
6.	KÄLLÉN, G., and WIGHTMAN, A.: The Analytic Properties of the Vacuum Expectation Value of a Product of three Scalar Local Fields. 1958	15.–
7.	BRODERSEN, SVEND, and LANGSETH, A.: The Fundamental Frequencies of all the Deuterated Benzenes. Application of the Complete Isotopic Rule to New Experimental Data. 1959	10.–
8.	MOTTELSON, BEN R., and NILSSON, SVEN GÖSTA: The Intrinsic States of Odd-A Nuclei having Ellipsoidal Equilibrium Shape. 1959	22.–
9.	KÄLLÉN, G., and WILHELMSSON, H.: Generalized Singular Functions. 1959	6.–
10.	MØLLER, C.: Conservation Laws and Absolute Parallelism in General Relativity. 1961	15.–
11.	SOLOVIEV, V. G.: Effect of Pairing Correlation on Energies and β-Transition Probabilities in Deformed Nuclei. 1961	8.–

Bind **2** (kr. 180.–)

1.	HIGGINS, JOSEPH: Theory of Irreversible Processes. I. Parameters of Smallness. 1962	17.–
2.	GALLAGHER, C. J., JR., and SOLOVIEV, V. G.: Two-Quasi-Particle States in Even-Mass Nuclei with Deformed Equilibrium Shape. 1962	18.–
3.	MANG, H. J., and RASMUSSEN, J. O.: Shell Model Calculations of Alpha Decay Rates of Even-Even Spheroidal Nuclei. 1962	12.–
4.	PELLEGRINI, C., and PLEBANSKI, J.: Tetrad Fields and Gravitational Fields. 1963	14.–
5.	NÖRLUND, N. E.: The Logarithmic Solutions of the Hypergeometric Equation. 1963	23.–
6.	LÜTKEN, HANS, and WINTHER, AAGE: Coulomb Excitation in Deformed Nuclei. 1964	9.–
7.	BARTLETT, JAMES H.: The Restricted Problem of Three Bodies. 1964	13.–
8.	VAN WINTER, CLASINE: Theory of Finite Systems of Particles. I. The Green Function. 1964	20.–
9.	GYLDENKERNE, KJELD: A Three-Dimensional Spectral Classification of G and K Stars. 1964	14.–
10.	VAN WINTER, CLASINE: Theory of Finite Systems of Particles. II. Scattering Theory. 1965	40.–

On direct application to the agent of the Academy, MUNKSGAARD, Publishers, 47 Prags Boulevard, København S., a subscription may be taken out for the series *Matematisk-fysiske Skrifter*. This subscription automatically includes the *Matematisk-fysiske Meddelelser* in 8vo as well, since the *Meddelelser* and the *Skrifter* differ only in size, not in subject matter. Papers with large formulae, tables, plates etc., will as a rule be published in the *Skrifter*, in 4to.

For subscribers or others who wish to receive only those publications which deal with a single group of subjects, a special arrangement may be made with the agent of the Academy to obtain the published papers included under one or more of the following heads: *Mathematics, Physics, Chemistry, Astronomy, Geology.*

In order to simplify library cataloguing and reference work, these publications will appear without any special designation as to subject. On the cover of each, however, there will appear a list of the most recent paper dealing with the same subject.

The last published numbers of *Matematisk-fysiske Skrifter* within the group of **Astronomy** are the following:

Vol. 3, nos. 1, 3.

Printed in Denmark
Bianco Lunos Bogtrykkeri A/S